

**6.6****Enrichment and Extension****Exploring Newton's Law of Cooling**

The natural base  $e$  is used to solve the equation that models Newton's Law of Cooling. The equation states that the difference in temperature between an object and its surroundings decreases exponentially as a function of time.

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

where  $T(t)$  = the temperature after  $t$  minutes,  $T_0$  = the initial temperature,  $T_s$  = the surrounding temperature,  $t$  represents time, and  $-k$  represents a constant rate of decrease in the temperature difference.

**Example:** When the constant surrounding temperature is  $65^\circ\text{F}$ , an object will cool from  $160^\circ\text{F}$  to  $120^\circ\text{F}$  in 30 minutes. After how long will the object's temperature be  $70^\circ\text{F}$ ?

**Solution:**

$$120 = 65 + (160 - 65)e^{-k(30)} \quad \text{Substitute the given values into the function.}$$

$$\frac{55}{95} = e^{-30k} \text{ is the same as } \frac{95}{55} = e^{30k} \quad \text{Simplify to solve for } k.$$

$$\ln\left(\frac{95}{55}\right) = \ln(e)^{30k} \rightarrow 0.5465 = 30k \rightarrow k = 0.0182$$

Use  $k$  to determine when the temperature is  $70^\circ\text{F}$ .

$$70 = 65 + (160 - 65)e^{-0.0182t}$$

$$\ln\left(\frac{5}{95}\right) = -0.0182t$$

$$t \approx 2 \text{ hours and } 42 \text{ minutes}$$

**In Exercises 1 and 2, use  $T(t) = T_s + (T_0 - T_s)e^{-kt}$  to solve the problem.**

- At a local restaurant, the cook prepares enough soup at night so that there is plenty of soup for customers the next day. Refrigeration is necessary, but the soup is too hot at  $220^\circ\text{F}$  to put directly into the fridge. It needs to be no more than  $70^\circ\text{F}$ . The cook cooled the soup by placing it in a sink of water constantly running at  $40^\circ\text{F}$ . After 10 minutes, the soup had cooled down to  $140^\circ\text{F}$ . After how long can the cook place the soup in the fridge?
- Milk is taken out of the fridge and left on a table. Its temperature is  $35^\circ\text{F}$  and the house is a constant  $70^\circ\text{F}$ . The temperature of the milk rose  $5^\circ\text{F}$  after 1 hour. However, milk is unsafe to consume at temperatures over  $45^\circ\text{F}$ . How long can the milk be left out and still be safe to consume?